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The Role of $\Delta(1232)$ in Two-pion Exchange Three-nucleon Potential

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Abstract

In this paper we have studied the two-pion exchange three-nucleon potential $(2\pi E - 3NP)$ using an approximate $SU(2) \times SU(2)$ chiral symmetry of the strong interaction. The off-shell pion-nucleon scattering amplitudes obtained from the Weinberg Lagangian are supplemented with the contributions from the well-known σ-term and the $\Delta(1232)$ exchange. It is the role of the Δ-resonance in $2\pi E - 3NP$, which we have investigated in detail in the framework of the Lagrangian field theory. The Δ-contribution is quite appreciable and, more significantly, it is dependent on a parameter Z which is arbitrary but has the empirical bounds $|Z| \leq 1/2$. We find that the Δ-contribution to the important parameters of the $2\pi E - 3NP$ depends on the choice of a value for Z, although the correction to the binding energy of triton is not expected to be very sensitive to the variation of Z within its bounds.

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1 Introduction

It is well-known that two-nucleon potentials are not always adequate in explaining nuclear properties. For example, all realistic two-body potentials which fit the two-nucleon data quite well, fail to reproduce the binding energy of triton [1, 2]. The experimental binding energy of ^3H is 8.48 MeV, while calculations with the well-known two-body local potentials fall short by 0.5 – 1.25 MeV. An obvious attempt to overcome the deficiency is to include the three-nucleon potential (3NP) in binding energy calculations. Computational techniques for trinuclear systems with the inclusion of three-nucleon potentials have become sufficiently mature to make such attempts worthwhile [1, 3]. Because of the short-range two-body repulsion between the nucleons tending to keep them apart, we expect that the two-pion exchange three-nucleon potential ($2\pi E - 3NP$) will have a larger effect than the relatively shorter range contributions to the 3NP due to the exchange of heavier mesons.

To construct the $2\pi E - 3NP$ we need the pion-nucleon scattering amplitudes with the pions off-mass-shell. An important mechanism in πN scattering is the formation of the Δ-resonance. We study the effect of the $\Delta(1232)$ by considering the most general form of the $\pi N\Delta$ interaction Lagrangian [4] which has been applied extensively in low energy πN scattering [5 - 8] and photo- and electro-production of pions [9]. This Lagrangian contains a parameter Z whose value is arbitrary. However, low energy phenomenology [5-9] constrains Z to lie between -1/2 and 1/2. The other pieces of the effective πN interaction Lagrangian have been obtained from the nonlinear chiral Lagrangian of Weinberg [10]. The Weinberg Lagrangian incorporates the nucleon-exchange effects in πN scattering and, in addition, there is either a direct $\pi\pi NN$ interaction, or πN scattering via ρ-exchange. Furthermore, we have added in the pion-nucleon σ-amplitude, $A^{(+)}_\sigma$, parametrized in an appropriate manner, to account for some well-known constraints [11 - 13] in the scattering amplitude $A^{(+)}$, which follow from Current Algebra and Partial Conservation of Axial-vector Current. The parameters of $A^{(+)}_\sigma$ have been adjusted by using the recent information on the amplitude $\bar{F}^{(+)}$ in the subthreshold region, obtained by analyzing the data from meson factories [14]. The model for pion-nucleon interaction so constructed is also compatible with low-energy πN data.

The two-pion exchange three-nucleon potential constructed from our model of πN interaction is dominated by the Δ-resonance and hence depends on Z. The nonlinear realization of chiral symmetry proposed by Weinberg [10] leads to a pseudovector πNN coupling which does not contribute to the $2\pi E - 3NP$ in the appropriate non-relativistic limits. The contribution to the $2\pi E - 3NP$ from the direct $\pi\pi NN$ interaction or from the ρ-exchange is small compared to the contribution from Δ-exchange.

Our purpose in this work is to examine in detail whether the parameter Z in the $\pi N\Delta$ interaction Lagrangian introduces appreciable Z-dependence in the three-nucleon potential and, consequently, in the calculations of physical quantities like the binding energy of triton. The three-nucleon potential obtained from our model is of the same form as the Tucson-Melbourne (TM) potential [15] or the Brazil poten-

tial [16], which contains four parameters a, b, c and d. In the present case b and d are functions of Z. The parameter b, which gives the dominant contribution to the binding energy correction of triton, is not very sensitive to Z, although the Δ-contribution to b, b_Δ, varies appreciably with Z. The reason for the insensitivity of b to variations of Z is that, in our model, the amplitude $A_\sigma^{(+)}$ is also indirectly Z-dependent through the slope parameter σ' (Sec. 3.3). The Δ-exchange and the amplitude $A_\sigma^{(+)}$ both contribute to the parameter b of the three-nucleon potential and their resultant contribution is such that b is more or less independent of Z. On the other hand, the parameter d varies appreciably with Z. However, the contribution from d to the binding energy correction B_3 of triton has been found to be much smaller compared with that from b [1, 3]. Therefore, the calculation of B_3 is not likely to be quite sensitive to the variation of Z as long as Z is constrained to lie within its empirical bounds, $|Z| \leq 1/2$.

The plan of the remaining portion of the paper is as follows: in Sec. 2 we review briefly the derivation of the $2\pi E - 3NP$ using the pion-nucleon off-shell scattering amplitudes as input, in Sec. 3 we discuss our model for pion-nucleon scattering and evaluate the nonrelativistic reduction of the amplitudes which are to be used in the $2\pi E - 3NP$. Finally, a detailed discussion of the results are given in Sec. 4.

2 Two-pion exchange three-nucleon potential

A three-nucleon potential means an irreducible potential energy function of the coordinates of the three nucleons — irreducible in the sense that the function cannot be written as a sum of functions involving fewer coordinates. The Feynman diagram corresponding to $2\pi E - 3NP$ is shown in figure 1. The amplitude for the process shown in figure 1 can be written as

$$< p_1' p_2' p_3' \mid S - 1 \mid p_1 p_2 p_3 >= -i\delta^4 \left(P - P'\right) \frac{1}{(2\pi)^5} \sqrt{\frac{m^6}{p_{10} p_{20} p_{30} p_{10}' p_{20}' p_{30}'}}\, T_{123}^{3N}, \quad (1)$$

where

$$T_{123}^{3N} = \left[\bar{u}(p_2')\gamma^\mu q_\mu \gamma_5 \tau_a u(p_2)\right] \frac{f/\mu}{q^2 - \mu^2} \left\{T_{\pi N}^{ba}\right\} \frac{f/\mu}{q'^2 - \mu^2} \left[\bar{u}(p_3')\gamma^\nu q_\nu' \gamma_5 \tau_b u(p_3)\right]. \quad (2)$$

The pseudovector coupling for the πNN vertex has been chosen in conformity with the results of the nonlinear realization of the chiral $SU(2) \times SU(2)$ symmetry for pion-nucleon interaction [10].

In the expressions (1) and (2) P and P' are the total four momenta before and after scattering; $q = p_2 - p_2'$ and $q' = p_3' - p_3$; a and b are the isospin indices of the pion, and μ is the mass. The off-shell pion-nucleon T-matrix $T_{\pi N}^{ba}$ describes the scattering process

$$\pi^a(q) + N(p_1) = \pi^b(q') + N(p_1'). \quad (3)$$

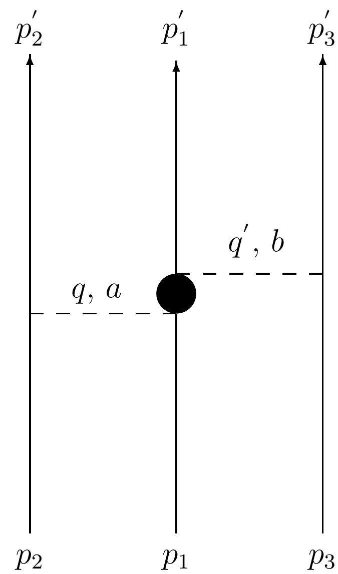

Figure 1: Two-pion exchange three-nucleon potential. The blob represents the pion-nucleon interaction

The pion-nucleon T-matrix is related to the S-matrix through the relation

$$< q'p_1' \mid S_{\pi N} - 1 \mid qp >= -i(2\pi)^4\delta^4\left(q + p_1 - q' - p_1'\right)\sqrt{\frac{m^2}{p_{10}p_{10}'q_0q_0'}}\, T_{\pi N}^{ba}(\nu, t, q^2, q'^2), \tag{4}$$

where ν and t are defined as

$$\nu = \frac{(q + q').(p_1 + p_1')}{4m}, \tag{5}$$

$$t = (q - q')^2. \tag{6}$$

The quantity ν is related to the Mandelstam variables $s = (q+p_1)^2$ and $u = (p_1 - q')^2$ by $\nu = (s - u)/4m$. Now the T-matrix $T_{\pi N}^{ba}$ has the general isospin decomposition

$$\begin{aligned} T_{\pi N}^{ba} \;=\; & \bar{u}(p_1')\left\{\left[A^{(+)} + \frac{1}{2}(\not{q} + \not{q}')B^{(+)}\right]\delta_{ba} + \right. \\ & \left. \left[A^{(-)} + \frac{1}{2}(\not{q} + \not{q}')B^{(-)}\right] i\epsilon_{bac}\tau_c\right\} u(p_1), \end{aligned} \tag{7}$$

where $A^{(\pm)}$ and $B^{(\pm)}$ are the isospin-even(+) and the isospin-odd(-) invariant amplitudes. An alternative isospin decomposition of $T_{\pi N}^{ba}$ is

$$T_{\pi N}^{ba} = \bar{u}(p_1')\left\{\left(F^{(+)} - \frac{[\,\not{q}', \not{q}\,]}{4m}B^{(+)}\right)\delta_{ba} + \left(F^{(-)} - \frac{[\,\not{q}', \not{q}\,]}{4m}B^{(-)}\right) i\epsilon_{bac}\tau_c\right\} u(p_1), \tag{8}$$

where

$$F^{(\pm)} = A^{(\pm)} + \nu B^{(\pm)}. \tag{9}$$

Now, since the potential is a nonrelativistic concept, we need to take the nonrelativistic limit of Eq. (1) to define a three-nucleon potential. In the theory of nonrelativistic potential scattering the S-matrix and the T-matrix are related by

$$< \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}' \mid s-1 \mid \vec{p}_1\vec{p}_2\vec{p}_3 >= -2\pi i\delta(E-E') < \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}' \mid t \mid \vec{p}_1\vec{p}_2\vec{p}_3 > . \tag{10}$$

In the first approximation the t-matrix in Eq. (10) can be equated to the three-nucleon potential W. The nonrelativistic reduction of Eq. (1) can be compared to Eq. (10) to obtain the 3NP. Thus we find

$$< \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}' \mid W(123) \mid \vec{p}_1\vec{p}_2\vec{p}_3 > \approx \; -\frac{1}{(2\pi)^6}\delta^3(\vec{P}-\vec{P}\,')t_{123}^{3N} , \tag{11}$$

where t_{123}^{3N} is the nonrelativistic reduction of T_{123}^{3N}. Taking the appropriate limit, we finally obtain

$$\begin{aligned}
&< \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}' \mid W(123) \mid \vec{p}_1\vec{p}_2\vec{p}_3 >= \\
&-\frac{1}{(2\pi)^6}\delta^3(\vec{P}-\vec{P}\,')\left(\frac{f}{\mu}\right)^2 \frac{H(\vec{q}\,^2)}{\vec{q}\,^2+\mu^2}\frac{H(\vec{q}\,'^2)}{\vec{q}\,'^2+\mu^2}(\vec{\sigma}_2.\vec{q})(\vec{\sigma}_3.\vec{q}\,')\tau_a^{(2)}\tau_b^{(3)} \times \\
&\left\{\left[f^{(+)} - \frac{i}{2m}\vec{\sigma}_1.(\vec{q}\times\vec{q}\,')b^{(+)}\right]\delta_{ba} + \left[f^{(-)} - \frac{i}{2m}\vec{\sigma}_1.(\vec{q}\times\vec{q}\,')b^{(-)}\right] i\epsilon_{bac}\tau_c^{(1)}\right\},
\end{aligned} \tag{12}$$

where $f^{(\pm)}$ and $b^{(\pm)}$ are the nonrelativistic limits of $F^{(\pm)}$ and $B^{(\pm)}$; $H(\vec{q}\,^2)$ and $H(\vec{q}\,'^2)$ refer to the form factors which are introduced because the pions are off-shell. We take $H(\vec{q}\,^2)$ as

$$H(\vec{q}^2) = \left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{q}\,^2}\right)^2 . \tag{13}$$

3 Model for pion-nucleon interaction

3.1 The Weinberg Lagrangian

We begin with the Weinberg Lagrangian [10] which is based on a nonlinear realization of the chiral $SU(2)\times SU(2)$ symmetry. The interaction Lagrangian relevant for pion-nucleon scattering can be written as

$$\mathcal{L}_W = \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN} \tag{14}$$

where

$$\mathcal{L}_{\pi NN} = (f/\mu)\bar{\psi}\gamma_5\gamma^\mu\tau_i\psi\partial_\mu\phi_i, \tag{15}$$

$$\mathcal{L}_{\pi\pi NN} = (i/4f_\pi^2)(\bar{\psi}i\gamma^\mu\tau_i\psi)\epsilon_{ijk}\phi_j\partial_\mu\phi_k. \tag{16}$$

Here ψ and ϕ are the nucleon and the pion fields, $f_\pi = 92.6$ MeV [17] is the pion decay constant and μ the mass of the pion. The interaction Lagrangian $\mathcal{L}_W$ consists of the usual derivative pion-nucleon coupling (Eq. 15) and a direct interaction between a pion and a nucleon (Eq. 16).

3.2 The $\pi N\Delta$ interaction Lagrangian

The most general form of the interaction Lagrangian $\mathcal{L}_{\pi N\Delta}$ can be written in the form [4]

$$\mathcal{L}_{\pi N\Delta} = \frac{1}{\sqrt{2}}\left(\frac{f^*}{\mu}\right)\left[i\overline{\Psi}_\mu \Theta^{\mu\nu} T_i \psi \partial_\nu \Phi_i + h.c.\right], \tag{17}$$

$$\Theta_{\mu\nu} = \left\{g_{\mu\nu} + \left[\frac{1}{2}(1+4Z)A + Z\right]\gamma_\mu\gamma_\nu\right\}, \tag{18}$$

where Ψ_μ is the Rarita-Schwinger field and the T's are a set of matrices corresponding to the isospin-$\frac{3}{2}$. The propagator for the $\Delta(1232)$ is written as

$$\langle 0|T(\psi_\mu(x)\bar{\psi}_\nu(y))|0\rangle = id_{\mu\nu}(\partial)\Delta_F(x-y) \tag{19}$$

where

$$\begin{aligned} &d_{\mu\nu}(\partial) = \\ &(i\gamma^\lambda\partial_\lambda + M)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3M}(\gamma_\mu i\partial_\nu - \gamma_\nu i\partial_\mu) + \frac{2}{3M^2}\partial_\mu\partial_\nu\right] + \\ &\frac{1}{3M^2}\left(\frac{A+1}{2A+1}\right)\left\{\left[-\frac{1}{2}\left(\frac{A+1}{2A+1}\right)i\gamma^\lambda\partial_\lambda + \left(\frac{A}{2A+1}\right)M\right]\gamma_\mu\gamma_\nu - \right. \\ &\left.\gamma_\mu i\partial_\nu - \left(\frac{A}{2A+1}\right)\gamma_\nu i\partial_\mu\right\}(\Box + M^2) \end{aligned} \tag{20}$$

and

$$\Delta_F(x-y) = \frac{1}{(2\pi)^4}\int d^4p\, \frac{exp[-ip(x-y)]}{p^2 - M^2 + i\epsilon}. \tag{21}$$

The interaction Lagrangian $\mathcal{L}_{\pi N\Delta}$ depends on two parameters A and Z. The parameter A, which occurs also in the propagator, can assume any value except -1/2 [4]. However, A drops out from the final expressions of the scattering amplitudes which therefore depend on Z only. There is no consensus on the exact value of Z, although $Z = 1/2$ is preferred theoretically [4]. From phenomenological studies a reliable bound,

$$|\ Z\ | \leq \frac{1}{2}, \tag{22}$$

can be placed on the value of Z [5 - 9]. The value of the $\pi N\Delta$ coupling constant is taken as ${f^*}^2/4\pi = 0.3359$. As the $\Delta(1232)$ makes the dominant contribution to the $2\pi E - 3NP$, and as there is some confusion regarding the magnitude of the $\Delta(1232)$ contribution, we shall discuss in detail this aspect of the problem in section 4.

3.3 The pion-nucleon σ-term

Current Algebra and PCAC impose certain constraints [11 - 13] on the isospin-even invariant amplitude $A^{(+)}$ at some unphysical values of the kinematical variables. To

satisfy these constraints and to account for the empirical information on the low-energy πN scattering, we include in our calculations an additional amplitude $A_\sigma^{(+)}$, called the pion-nucleon σ-amplitude which is parametrized [6 - 8, 18] as follows:

$$A_\sigma^{(+)}(\nu, \nu_B) = \frac{\sigma_{NN}(t = 2\mu^2)}{f_\pi^2} \left[\frac{q^2 + q'^2 - \mu^2}{\mu^2} + \frac{\sigma'(4m\nu_B)}{\mu^2} \right], \tag{23}$$

where

$$\nu = (s - u)/4m, \;\; \nu_B = (t - q^2 - q'^2)/4m;$$

s, t and u are the Mandelstam variables, q and q' are the momenta of the incoming and the outgoing pion respectively.

Recently the pion-nucleon scattering amplitudes in the subthreshold region have been recalculated by using the meson factory πN data and dispersion relation [14]. The important results relevant for our discussion are

$$\begin{array}{lcr} \bar{F}^{(+)}(\nu = 0,\, t = 2\mu^2,\, q^2 = \mu^2,\, q'^2 = \mu^2) & \approx & 1.35\,\mu^{-1} \\ \bar{F}^{(+)}(\nu = 0,\, t = \mu^2,\, q^2 = \mu^2,\, q'^2 = \mu^2) & \approx & -0.08\,\mu^{-1} \\ \bar{F}^{(+)}(\nu = 0,\, t = 0,\, q^2 = \mu^2,\, q'^2 = \mu^2) & \approx & -1.34\,\mu^{-1} \end{array} \tag{24}$$

which yield

$$\sigma_{NN}(t = 2\mu^2) = 82\,\mathrm{MeV} \tag{25}$$

and

$$\sigma' = \begin{cases} 0.66 & \text{for Z = 1/2} \\ 0.50 & \text{for Z = 1/4} \\ 0.40 & \text{for Z = 0} \\ 0.36 & \text{for Z = - 1/4} \\ 0.37 & \text{for Z = -1/2 .} \end{cases} \tag{26}$$

The amplitude $\bar{F}^{(+)}$ is the remainder of $F^{(+)}$ after the (pseudovector) nucleon Born terms have been subtracted from it. It may be noted here that while the value of σ' is sensitive to the choice of the parameter Z in the $\pi N \Delta$ interaction Lagrangian, $\sigma_{NN}(t = 2\mu^2)$ is independent of Z.

3.4 Nonrelativistic limits of pion-nucleon scattering amplitudes

The contributions to the pion-nucleon invariant amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ due to nucleon-exchange, Δ-exchange and direct $\pi\pi NN$ interaction can be easily calculated and are quoted in different places [4, 19]. For our purpose we need only the nonrelativistic reductions $f^{(\pm)}$ and $b^{(\pm)}$ of the amplitudes $F^{(\pm)} = A^{(\pm)} + \nu B^{(\pm)}$ and $B^{(\pm)}$ respectively.

First, consider the nucleon-exchange contribution to pion-nucleon scattering. The invariant amplitudes $A_N^{(\pm)}$ and $B_N^{(\pm)}$ consists of the forward propagating Born term (FPBT) and the backward propagating Born term (BPBT). The FPBT is already

accounted for as the iterate of two-nucleon one-pion exchange potential. Therefore the FPBT has to be subtracted from the invariant amplitudes. If we take the nonrelativistic limit of what remains we obtain

$$f_N^{(\pm)} = 0 \ , \ \ b_N^{(\pm)} = 0 \, . \tag{27}$$

We thus see that the nucleon-exchange contribution to the πN amplitudes does not contribute to the $2\pi E - 3NP$. The results in Eq. (27) are correct only if we use the gradient coupling for πNN interaction.

For the Δ-contribution to the πN amplitudes, we find

$$\begin{aligned} F_\Delta^{(+)} \rightarrow \ \ f_\Delta^{(+)} &= \alpha_\Delta^{(+)} \vec{q}.\vec{q}\,', \\ F_\Delta^{(-)} \rightarrow \ \ f_\Delta^{(-)} &= 0, \end{aligned} \tag{28}$$

and

$$\begin{aligned} B_\Delta^{(+)} \rightarrow \ \ b_\Delta^{(+)} &= 0, \\ B_\Delta^{(-)} \rightarrow \ \ b_\Delta^{(-)} &= \beta_\Delta^{(-)}(Z), \end{aligned} \tag{29}$$

where

$$\begin{aligned} \alpha_\Delta^{(+)} = \ & \left(2f^{*^2}/9\mu^2\right) \left[\frac{(4M^2 - Mm + m^2)}{(M-m)M^2} \right. \\ & \left. - \frac{4(M+m)Z}{M^2} - \frac{4(2M+m)Z^2}{M^2} \right] \end{aligned} \tag{30}$$

and

$$\begin{aligned} \beta_\Delta^{(-)} = \ & \left(f^{*^2}/9\mu^2\right) \left[\frac{2m(2M^2 + Mm - m^2)}{(M-m)M^2} \right. \\ & \left. + \frac{8m(M+m)Z}{M^2} + \frac{8m(2M+m)Z^2}{M^2} \right] . \end{aligned} \tag{31}$$

Here $M = 1232$ MeV is the mass of $\Delta(1232)$ and $m = 938.9$ MeV is the nucleon mass. The quantities $\alpha_\Delta^{(+)}$ and $\beta_\Delta^{(-)}$ are obtained from the expressions for the Δ-contribution to the amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ as given in Ref. [19]. These results (Eqs. 30, 31) are the same as derived earlier by Coelho, Das and Robilotta [16]. However, they chose $Z = -1/2$ for the detailed discussions on the Δ-contribution to the three-nucleon potential.

Next, for the direct $\pi\pi NN$ interaction, only the amplitude $B_d^{(-)} = 1/2f_\pi^2$ is nonzero. We therefore have

$$b_d^{(+)} = 0, \ \ b_d^{(-)} = 1/2f_\pi^2, \tag{32}$$

and

$$f_d^{(+)} = 0, \ \ f_d^{(-)} = \nu/2f_\pi^2 \approx 0. \tag{33}$$

Finally, the nonrelativistic limit of the σ-contribution to the πN amplitude is also simple. We have

$$f_\sigma^{(+)} = a_\sigma^{(+)} = \frac{\sigma_{NN}(t=2\mu^2)}{f_\pi^2}\left[-1 + \frac{2\sigma'\vec{q}.\vec{q}\,'}{\mu^2} - \frac{(\vec{q}^{\,2}+\vec{q}\,'^{\,2})}{\mu^2}\right]. \qquad (34)$$

Since the pion is off-shell, each of the pion-nucleon amplitudes have to be multiplied by the form factor given in Eq. (13). Now, inserting the separate contributions to $f^{(\pm)}$ and $b^{(\pm)}$ in Eq. (12) we can write the $2\pi E - 3NP$ as

$$\begin{aligned}
&< \vec{p}_1{}'\vec{p}_2{}'\vec{p}_3{}' \mid W(123) \mid \vec{p}_1\vec{p}_2\vec{p}_3 >= \\
&\quad \frac{1}{(2\pi)^6}\delta^3(\vec{P}-\vec{P}\,')\left(\frac{f}{\mu}\right)^2 \frac{H(\vec{q}^{\,2})}{(\vec{q}^{\,2}+\mu^2)}\frac{H(\vec{q}\,'^{\,2})}{(\vec{q}\,'^{\,2}+\mu^2)}(\vec{\sigma}_2.\vec{q})(\vec{\sigma}_3.\vec{q}\,')\times \\
&\quad \left\{\vec{\tau}_2.\vec{\tau}_3\left[a + b\vec{q}.\vec{q}\,' + c(\vec{q}^{\,2}+\vec{q}\,'^{\,2})\right] - d(\vec{\tau}_1.\vec{\tau}_2\times\vec{\tau}_3)(\vec{\sigma}_1.\vec{q}\times\vec{q}\,')\right\}, \qquad (35)
\end{aligned}$$

where

$$\begin{aligned}
a &= \sigma_{NN}(t=2\mu^2)/f_\pi^2 \\
b &= -\alpha_\Delta^{(+)}(Z) - 2\sigma'\frac{\sigma_{NN}(t=2\mu^2)}{f_\pi^2\mu^2} \\
c &= \frac{\sigma_{NN}(t=2\mu^2)}{f_\pi^2\mu^2} \\
d &= -\frac{\beta_\Delta^{(-)}(Z)}{2m} - \frac{1}{4mf_\pi^2}\,. \qquad (36)
\end{aligned}$$

The $2\pi E - 3NP$ given in Eq. (35) is of the same form as that derived by the Tucson-Melbourne (TM) group [15] except that the coefficients a, b, c and d in the TM potential are:

$$\begin{aligned}
a &= 1.130\,\mu^{-1} \\
b &= -2.580\,\mu^{-3} \\
c &= 1.000\,\mu^{-3} \\
d &= -0.753\,\mu^{-3}.
\end{aligned} \qquad (37)$$

4 Results and conclusions

The parameters a, b, c and d in Eqs. (36) receive contributions from $A_\sigma^{(+)}$, the Δ-exchange and the direct term for πN scattering. The Δ-exchange contributes to b and d, while the direct πN interaction only to d. The parameters a and c receive contributions from $A_\sigma^{(+)}$ only; $A_\sigma^{(+)}$ contributes also to b.

In order to show the relative importance of the various contributions, we refer to table 1 where we have shown the values of a, b, c and d corresponding to five different values of Z, namely $Z = 1/2$, 1/4, 0, -1/4 and -1/2. Note that a and c are independent of Z, but b and d are not. Regarding the parameter b, as Z is decreased from 1/2 to -1/4, the contribution b_Δ from the Δ-exchange decreases, while the contribution

b_σ from $A_\sigma^{(+)}$ increases. However, this trend is reversed somewhat at Z = -1/2. As a result, the net b is not very sensitive to the variation of Z within its acceptable bounds, $|Z| \leq 1/2$. Note that the Z-dependence of b_σ is due to σ' which depends on Z (Eq. 26). Next, the parameter d receives a small Z-independent contribution from the direct πN scattering term, while the dominant contribution to d comes from the Δ-exchange which depends on Z. The value of d increases steadily as Z is decreased from 1/2 to -1/4, then it decreases slightly at Z = -1/2 (table 1).

Also shown in table 1 is the ratio b_Δ/d_Δ which ranges from 1.26 to 4.0 as Z is varied from 1/2 to -1/2. This contradicts the often quoted result that b_Δ/d_Δ should be equal to four as a rule [20]. In our calculations this ratio is four only if $Z = -1/2$. However, there is no *a priori* justification for choosing this value of Z. In fact, in the theory of spin-3/2 field [4, 7], Z = -1/2 corresponds to calculations with a $\pi N\Delta$ vertex and a Δ-propagator taking the Δ on-mass-shell in both cases. More explicitly, if we take $A = -1$ in Eqs. (18, 20) and then $Z = -1/2$ in Eq. (18), the off-mass-shell parts of Δ are eliminated from both the propagator and the interaction Lagrangian. Peccei [21] obtained a special form for the interaction Lagrangian $\mathcal{L}_{\pi N\Delta}$ which would correspond to $Z = -1/4$ in our formalism. For this value of Z, the ratio b_Δ/d_Δ is 4.23. However, if we take $Z = 1/2$, the theoretically preferred value [4], this ratio is 1.26, much smaller than 4. It has already been noted in section (3.2) that, while A may be assigned any value except $A = -1/2$, the empirical bounds on Z is $\mid Z \mid \leq 1/2$.

Our main purpose in this paper is to see in what way and to what extent the Z-dependence of the $\pi N\Delta$ interaction Lagrangian effects the two-pion-exchange three-nucleon potential and whether any sensitive Z-dependence is likely to appear in calculations of relevant physical quantities, for example, the binding energy of triton.

A first-order perturbation calculation for the correction E_3 to the energy of triton due to the Tucson-Melbourne potential with the parameters a, b, c and d as in Eqs. (37) was done by Ishikawa *et al* [3]. The zeroth-order triton wave function was obtained by solving the Faddeev equations with a variety of two-nucleon potentials. Ishikawa *et al* used the dipole form factor at the vertices with several values of the cut-off parameter Λ. For Λ = 800 MeV and the Reid soft-core two-nucleon potential they found that the contributions to E_3 from the individual terms of the TM potential corresponding to the parameters a, b, c, and d (Eqs. 37) are 0.05 MeV, -0.97 MeV, 0.25 MeV and -0.22 MeV respectively. In the first-order calculations of Ishikawa *et al*, E_3 is linear in a, b, c and d. Since the two-pion exchange three-nucleon potential in our model is of the same form as the TM potential, we can easily estimate the binding energy correction $B_3(= -E_3)$ for triton due to our $\pi\pi$-exchange three-nucleon potential, simply by scaling Ishikawa *et al*'s results (table 2). This is done solely to examine the possible Z-dependence of B_3, although a first-order perturbative calculation is not expected to be very accurate. We see that as Z decreases from 1/2 to -1/2, B_3^Δ increases initially and then remains more or less constant for negative Z, while B_3^σ decreases and then becomes negligible for $Z \leq 0$. Therefore, $B_3 = B_3^\Delta + B_3^\sigma + B_3^d$, is not very sensitive to the variation of Z. This conclusion would not change appreciably if the direct πN interaction is replaced by the ρ-mediated interaction.

It may be noted here that the numerical values for B_3 will depend on the choice

Table 1: The parameters a, b, c and d in the two-pion exchange three-nucleon potential displayed for five values of Z within the bounds $|Z| \leq 1/2$. Note that a and c are independent of Z.

$a = a_\sigma$ (μ^{-1})	$c = c_\sigma$ (μ^{-3})	Z	b_Δ (μ^{-3})	b_σ (μ^{-3})	b (μ^{-3})	d_Δ (μ^{-3})	d_d (μ^{-3})	d (μ^{-3})	b_Δ/d_Δ
1.341	1.341	1/2	-1.038	-1.770	-2.808	-0.821	-0.056	-0.877	1.26
		1/4	-1.445	-1.341	-2.786	-0.618		-0.674	2.34
		0	-1.706	-1.073	-2.779	-0.487		-0.543	3.50
		-1/4	-1.820	-0.966	-2.786	-0.430		-0.486	4.23
		-1/2	-1.787	-0.992	-2.779	-0.447		-0.503	4.0

Table 2: The correction, $B_3 = B_3^\Delta + B_3^\sigma + B_3^d$, to the binding energy of triton due to the two-pion exchange three-nucleon potential. The contributions B_3^Δ and B_3^σ are Z-dependent. The table also shows the binding energy B_2 of triton for the two-nucleon Reid soft-core potential, the total binding energy $B = B_2 + B_3$ and $B_{exp} - B$. All the binding energies are in MeV.

B_2	B_3^d	Z	B_3^Δ	B_3^σ	B_3	B	B_{exp}	$B_{exp} - B$
7.24	0.02	1/2	0.63	0.27	0.92	8.16	8.48	0.32
		1/4	0.72	0.11	0.85	8.09		0.39
		0	0.78	0.01	0.81	8.05		0.43
		-1/4	0.81	-0.03	0.80	8.04		0.44
		-1/2	0.80	-0.02	0.80	8.04		0.44

of the two-nucleon potential and the value of the cut-off parameter Λ. In particular, the binding energy correction is quite sensitive to Λ [1, 3]. The dependence on Λ may be reduced somewhat if one includes the $\rho\pi$-exchange, $\rho\rho$-exchange three-nucleon forces in addition to the $\pi\pi$-exchange three-nucleon force [1]. However, our purpose is to investigate in detail the effect of $\Delta(1232)$ on the parameters of the two-pion exchange three-nucleon potential. The resonance $\Delta(1232)$ makes large contributions to the parameters b and d, and these contributions depend on Z. We find that b_Δ is sensitive to the variation of Z, although $b = b_\Delta + b_\sigma$ does not change appreciably with Z. The parameter d, however, depends substantially on Z.

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